Laplace Plane Analysis of Skin Impedance: A Preliminary Investigation

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ABSTRACT

The impedance and phase angle of skin as functions of frequency were determined by Laplace plane analysis of the time domain current resulting from an external voltage perturbation. Bode plots of these functions established a passive equivalent circuit model for skin impedance which is valid over a wide range of frequencies. Typical values for the model circuit elements are given for ten human subjects.

The electrical properties of any biological tissue depend on its intrinsic structure and, for *in vivo* studies, on the functional state of the organism. For the case of human skin, the impedance can vary with the thickness and moisture content of the stratum corneum, the concentration and activity of sweat glands, localized injury, the age of the subject, and environmental factors such as temperature and humidity. Seasonal and diurnal variations have been measured: For example, the d-c resistance of skin tends to increase during sleep. Short-term fluctuations are the basis for various monitoring techniques, including the electrocardiogram, electroencephalogram, and impedance plethysmography. Changes in impedance are also associated with the psychological state of the subject (1).

Studies of the a-c impedance of biological systems, including human skin, have generally involved direct measurement of the impedance and phase angle as functions of the frequency of the applied voltage or current (2-5). Any such measurement over a wide range of frequencies tends to be cumbersome and time-consuming. Measurements with an impedance bridge have the additional disadvantage that some *a priori* assumptions must be made regarding an equivalent circuit for the skin impedance. Other reports have presented approximation methods for deriving the equivalent circuit components from the current response to a square voltage pulse input (6, 7). Burton (8) applied Bode analysis to measurements of the skin impedance and phase angle. By means of this method a passive equivalent circuit can be synthesized for any electrical "black box" from plots of its impedance and phase angle *vs.* frequency. The only assumption required is that the system consists solely of passive linear lumped circuit elements (8, 9). Even though the resulting model is not necessarily unique (8), it must represent the system accurately in the frequency range studied.

By extending this technique one step further, the entire frequency spectrum of any system can be computed by applying a Laplace transformation to the current response to an arbitrary voltage perturbation, thus eliminating the tedious process of point-by-point measurement of the impedance and phase angle.

Method

In order to obtain the impedance of a system as a function of frequency, the input voltage perturbation $V(t)$ and the resulting current $I(t)$ must be converted from the time domain (t) to the complex frequency domain (s). This may be accomplished by means of a Laplace transformation. The transform $\mathbf{F}(s)$ of a time domain function $f(t)$ is defined as

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$$\mathbf{F}(s) = \int_0^\infty f(t) \exp(-st)\,dt \qquad [1]$$

where $s = \sigma + j\omega$ denotes the complex frequency plane with real axis σ and imaginary axis $j\omega$ (9-11). The integration (1) can be carried out along either the real or imaginary axis of the complex frequency plane. Considering the real axis $(s = \sigma)$

$$\mathbf{F}(\sigma) = \int_0^\infty f(t) \exp(-\sigma t)\,dt, \quad \sigma > 0 \qquad [2]$$

This integral exists for any converging function $f(t) \exp(-\sigma t)$. Performing real axis transformations on both $V(t)$ and $I(t)$, we can define the real axis impedance of the system as

$$Z(\sigma) = V(\sigma)/I(\sigma) \qquad [3]$$

The analogous imaginary axis Laplace transformation $(s = j\omega)$ is given by

$$\mathbf{F}(j\omega) = \int_0^\infty f(t) \exp(-j\omega t)\,dt \qquad [4]$$

equivalent to a one-sided Fourier transform. The corresponding imaginary axis impedance function is

$$Z(j\omega) = V(j\omega)/I(j\omega) \qquad [5]$$

$Z(j\omega)$ is a complex function with a real component $\mathrm{Re}Z(\omega)$ and an imaginary component $\mathrm{Im}Z(\omega)$ which define the phase angle

$$\phi(\omega) = \tan^{-1}[\mathrm{Im}Z(\omega)/\mathrm{Re}Z(\omega)] \qquad [6]$$

A FORTRAN program for both real and imaginary axis Laplace transformations has been developed by Pilla (10, 11). The input data are points defining $V(t)$, an arbitrary voltage perturbation, and $I(t)$, the current response of the system under observation from time $t = 0$ to the time at which $I(t)$ reaches its d-c limit. The program is written for a $V(t)$ which increases from zero to some final constant value and an $I(t)$ rising from zero to a single maximum before decreasing to its d-c limit. We found that the subject current matched these constraints. The output data include $Z(\sigma)$, $\mathrm{Re}Z(\omega)$, $\mathrm{Im}Z(\omega)$, and $\phi(\omega)$ as functions of frequency over any desired range. The highest attainable frequency range is proportional to the reciprocal of the smallest time after $t = 0$ at which both $V(t)$ and $I(t)$ are measurable. The Laplace transform method of obtaining these functions is particularly advantageous in that $Z(\sigma)$ and $Z(j\omega)$ are theoretically independent of the properties of the measuring electrodes, provided that these are as nearly identical as possible (11).

Experimental Procedure

The voltage $V(t)$, a pulse with a rise time of 10 μsec, a duration of 100 μsec, and a maximum amplitude of 1V, was displayed on one channel of a Tektronix 564 dual-trace oscilloscope; the subject current $I(t)$ was displayed on the second channel (see Fig. 1). Preliminary studies indicated that a pulse duration of 100 μsec was sufficiently long to establish the d-c current limit for all subjects. The interval was set at 1000 μsec (10 times the pulse duration) throughout the study. The resulting display was adequate for photography at sweep rates slower than 0.2 μsec/cm. After the skin has been cleaned with 90% ethanol and hydrated with distilled water, two 1 cm diam carbon-impregnated conducting-rubber electrodes (modified L1DC electrodes, Ritter Company) were applied to the dorsal surfaces of the hand and upper forearm, both in areas devoid of cuts, abrasions, or pigmented moles. We found that the results obtained with these electrodes were comparable to those with metal electrodes, with the advantages of greater flexibility and ease of application. In order to study the skin in as physiological a state as possible, no electrode paste was used. The ensuing oscilloscope display of $V(t)$ and $I(t)$ was photographed for later analysis; typical curves are shown in Fig. 2. Several exposures at sweep speeds ranging from 0.5 to 10 μsec/cm were needed to obtain sufficient data for subsequent mathematical analysis. This procedure was repeated on a total of 10 volunteer subjects.

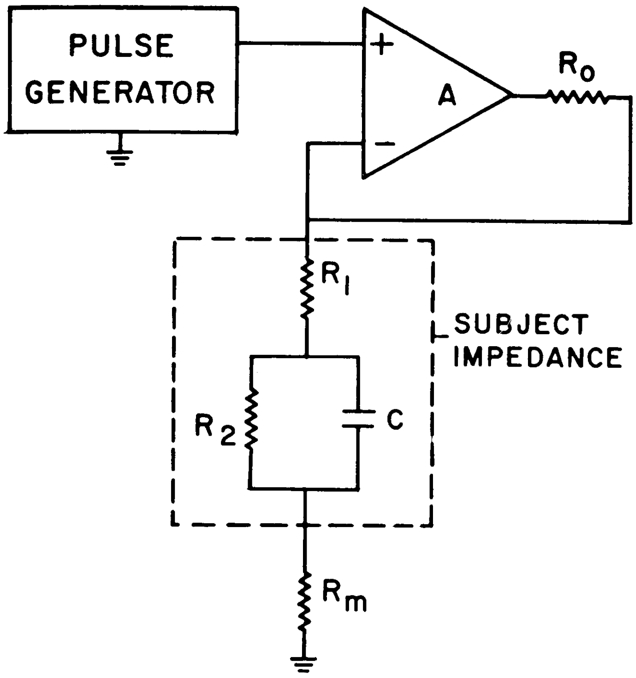

Fig. 1. Potentiostatic circuit for the study of skin impedance. The power supply consisted of a Tacussel model PRT-20-010-MOD potentiostat (A) with output impedance R_o, controlled by a model GSTP-10 pulse generator, applied to the subject. The voltage *V(t)*, a pulse with a rise time of 10 μsec and a duration of 100 μsec, was displayed on one channel of a Tektronix 564 storage oscilloscope with a 3A6 dual-trace input and 3B4 time base. The subject current response *I(t)*, proportional to the voltage across a small series resistor R_m, was displayed on the second channel. The model shown for the subject is a standard equivalent circuit for the a-c impedance of human skin (1,3,6,8).

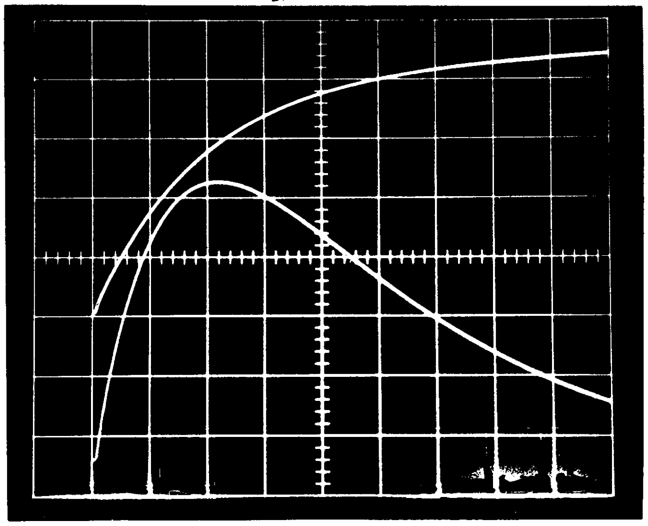

Fig. 2. Typical response curves at 2 μsec/cm. The upper trace is *V(t)* at 0.2 V/cm; the lower trace is *I(t)* at 5 μA/cm.

Points defining $V(t)$ and $I(t)$, taken from the photographs, were used as the input time domain data for real and imaginary axis Laplace transformations performed for each individual test by means of a FORTRAN program for numerical integration (10, 11). To attain an upper frequency limit of 1 MHz, it was necessary to record $V(t)$ and $I(t)$ from $t = 0.1$ μsec, with no more than a 10% variation in either parameter between successive data points. These conditions required 50-70 such data points per test. The output of the program included the phase angle and real and imaginary components of the impedance as functions of frequency (d-c to 1 MHz).

Results

Analysis of the frequency domain data for human skin resulting from the real and imaginary axis Laplace transformations established that all of the impedance *vs.* frequency curves were of the same form, and could be described by the real impedance function

$$Z(\sigma) = R_1 + 1/(\sigma C + 1/R_2) \qquad [7]$$

This corresponds to a simple *R-C* circuit (see Fig. 1) found by Burton (8) to be an adequate representation of the skin impedance. Similarly, the data from the imaginary axis transformation yielded the complex impedance

$$Z(j\omega) = R_1 + 1/(1/R_2 + j\omega C) \qquad [8]$$

$Z(j\omega)$ could be separated into its real and imaginary components

$$\mathrm{Re}Z(\omega) = R_1 + 1/(\omega^2C^2 + 1/R_2^2) \qquad [9]$$

and

$$\mathrm{Im}Z(\omega) = -\omega C/(\omega^2C^2 + 1/R_2^2) \qquad [10]$$

with a phase angle

$$\phi(\omega) = \tan^{-1}(\omega C/[1 + R_1(\omega^2C^2 + 1/R_2^2)] \qquad [11]$$

Once the form of the equivalent circuit was established, it was not necessary to repeat the entire analytical procedure for each individual test. When analyzing large volumes of data, the model circuit components (R_1, R_2, and C) could be derived from the appropriate low and high frequency limits of these functions (10, 11). These are listed in Table I.

Table I. Low ($R_2\omega C << 1$) and high ($R_2\omega C >> 1$) frequency limits of $Z(\sigma)$, $ReZ(\omega)$, $ImZ(\omega)$, and $\tan\phi(\omega)$

	$R_2\omega C << 1$	$R_2\omega C >> 1$
$Z(\sigma)$	$R_1 + R_2$	$R_1 + 1/\sigma C$
$ReZ(\omega)$	$R_1 + R_2$	$R_1 + 1/R_2\omega^2C^2$
$ImZ(\omega)$	$-\omega CR_2^2$	$-1/\omega C$
$\tan\phi(\omega)$	$-\omega CR_2^2/(R_1 + R_2)$	$-1/\omega CR_1$

In this case, the low frequency limit of both $Z(\sigma)$ ($\sigma CR_2 << 1$) and $ReZ(\omega)$ ($\omega CR_2 << 1$) is $R_1 + R_2$. $|ImZ(\omega)|$ *vs.* ω is a straight line through the origin with slope CR_2^2. At high frequencies ($\sigma CR_2 >> 1$; $\omega CR_2 >> 1$) $|ImZ(\omega)|$ *vs.* $1/\omega$ becomes a straight line with slope $1/C$; $Z(\sigma)$ and $ReZ(\omega)$ are lines with intercepts R_1 and slopes $1/C$. Sample curves are given in Fig. 3. All the equivalent circuit elements can be determined from the results of either transform: it is not necessary to the analysis to perform both real and imaginary axis transforms.

Values of R_1, R_2, C, and the minimum angle ϕ_{min} for all 10 subjects are listed in Table II. These results are compatible with those of earlier studies. As noted by Plutchik (4), the phase angle apparently varied much less between subjects than did the other parameters (see Table II). The equivalent circuit is valid for all anatomical locations. The actual values of the circuit elements would, however, vary with location for each individual. For example, the d-c resistance ($R_1 + R_2$) of the dorsum of the hand tends to be higher than that of the palm (1). R_2 represents the d-c resistance of the stratum corneum, probably separated by a discrete basal cell membrane from the inner tissue resistance R_1. An injury, such as a pinprick, results in a significant drop in the measured

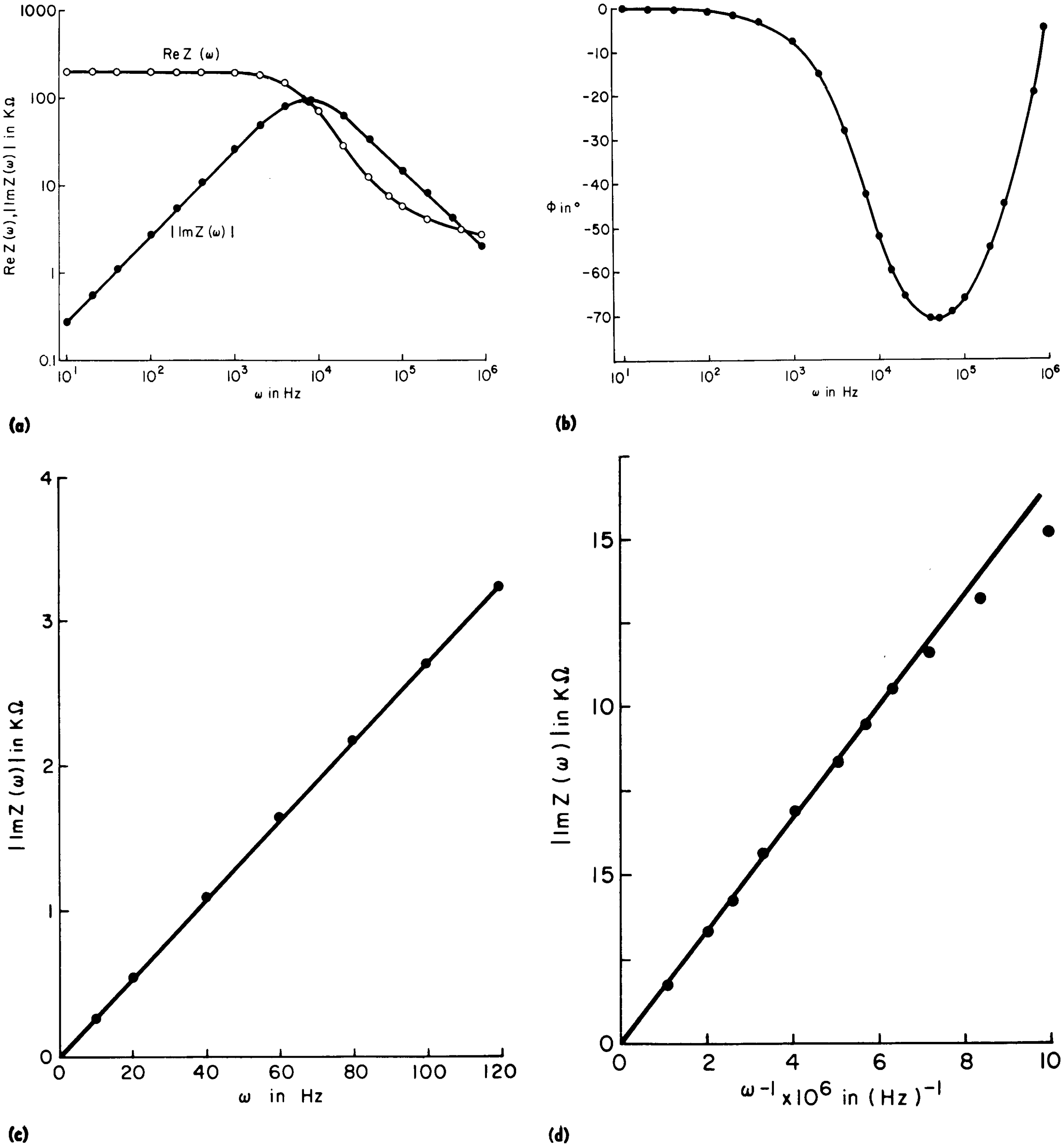

Fig. 3. Typical results of an imaginary axis Laplace transformation. (a) $ReZ(\omega)$ *vs.* ω and $|ImZ(\omega)|$ *vs.* ω; (b) $\phi(\omega)$ *vs.* ω; (c) $|ImZ(\omega)|$ *vs.* ω at low frequencies, ($R_2\omega C << 1$) has slope CR_2^2 and intercept 0; (d) $|ImZ(\omega)|$ *vs.* $1/\omega$ at high frequencies, ($R_2\omega C >> 1$) has slope $1/C$.

Table II. Mean values of R_1, R_2, C, and ϕ_{min} for each of 10 subjects

Subject	R_1 (KΩ)	R_2 (KΩ)	C (pf)	ϕ_{min} (deg.)
1	17.8 ± 4.1	768 ± 299	298 ± 15.6	−72.7 ± 1.3
2	2.06 ± 0.08	159 ± 126	1730 ± 226	−75.3 ± 6.0
3	17.5 ± 0.6	292 ± 34	252 ± 2.8	−63.2 ± 1.8
4	29.9 ± 0.9	561 ± 50	165 ± 3.5	−64.6 ± 1.4
5	6.51 ± 1.80	344 ± 14	430 ± 45	−73.0 ± 7.5
6	6.19 ± 0.09	186 ± 14	720 ± 18	−69.7 ± 0.9
7	6.72 ± 0.40	224 ± 28	177 ± 15	−70.6 ± 1.7
8	212 ± 2.1	1900 ± 156	47.7 ± 0.3	−54.9 ± 1.3
9	199 ± 15	2480 ± 240	35.8 ± 4.6	−59.5 ± 0.3
10	5.89 ± 0.4	112 ± 9.9	893 ± 13	−64.8 ± 0.3

d-c resistance in that area, followed by a return to the normal level during healing. It is at present unclear whether C is a structural or merely a polarization capacitance. If the former, it could be determined by the dielectric properties of skin in conjunction with the detailed structure of the epidermis (1).

Conclusion

Data on the time domain current response of human skin to an external voltage perturbation have been subjected to a Laplace transformation, yielding the impedance and phase angle in the frequency domain. Subsequent Bode analysis of this data provided a simple equivalent circuit for skin impedance, identical to a standard model for the steady-state impedance.

This technique could be useful for the study of the frequency response of any biological system which can be described by an equivalent circuit composed of passive, linear, and lumped circuit elements. Pilla (10) has noted the desirability of automating the data-recording process for this technique. If indeed this should prove feasible, the technique of frequency domain analysis would greatly extend the study of transient responses, at present limited to monitoring of resistance or potential levels.

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